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DISCUSSION OF FREQUENCY ANALYSIS OF BEAM AND GIRDER FLOORS

(Published in October, 1949)

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DISCUSSION

ROBERT V. HAUER.[6]—This procedure for finding the natural frequencies of floor systems (or any other combination of structural members) will be of great value in all cases in which it is imperative to avoid the resonance of machines not only with the fundamental mode, but also with the higher harmonics of the structure. In buildings, however, such cases are comparatively rare. Usually, the designer will be concerned with only the gravest frequency, because the amplitudes of the higher modes are generally kept in check by the ever-present damping.

Whenever only the frequency of the fundamental mode is required, a shorter way of arriving at the solution is by equating the maximum potential and kinetic energies, as developed by Lord Rayleigh.[7] Although this method gives only approximate results (because a suitable assumption has to be made concerning the shape of the structure during vibration), it is known from all cases in which the exact and the approximate solutions could be compared that the accuracy of Lord Rayleigh's method is surprisingly high and more than sufficient for all practical purposes.

In applying the energy method to a floor system, the problem of how to correlate the shapes of the different members arises. This difficulty can be easily overcome by determining the energies throughout from the shapes of the statical deflections. Moreover, this particular choice leads to an extremely simple formula for the fundamental circular frequency[8]—namely,

$$\omega = \sqrt{g \frac{\Sigma P \, \delta}{\Sigma P \, \delta^2}} \quad \ldots\ldots\ldots\ldots (86)$$

in which g is the acceleration of gravity; P represents the loads; and δ is the corresponding deflections. According to its derivation, Eq. 86 can be applied to any system and will appeal especially to structural engineers, because it reduces the vibration problem essentially to the routine task of computing deflections.

The writer welcomed the opportunity to check the accuracy of the energy method in the case of a floor system, for which Mr. Bleich has provided the exact solution. Applied to the numerical example of the paper, Eq. 86 yields a fundamental frequency of 41.1 per sec. This is only 1% more than the correct frequency value of 40.7 per sec—that is, practically equivalent.

NOTE.—This paper by Hans H. Bleich was published in October, 1949, *Proceedings*. The numbering of footnotes, equations, and illustrations in this Separate is a continuation of the consecutive numbering used in the original paper.

[6] Structural Engr., Albert Kahn, Associated Archts. and Engrs., Inc., Detroit, Mich.

[7] "Theory of Sound," by Lord Rayleigh, Macmillan & Co., New York, N. Y., 2d Ed., 1877.

[8] "Vibration Problems in Engineering," by S. Timoshenko, D. Van Nostrand Co., Inc., New York, N. Y., 2d Ed., 1937, pp. 94–95.

As the floor layout shown in Fig. 12 is symmetrical about the axis *b–b*, it follows by inspection that the frequencies of some higher modes of the entire system must coincide with the fundamental frequencies of the individual beams T_2 (or T_3, T_6, T_7) and T_1 (or T_4, T_5, T_8). In this particular case, therefore, these higher frequencies can also be computed by Eq. 86. The results are as follows: $\omega_2 = 54.3$ (from T_2); and $\omega_4 = 94.3$ (from T_1). These values compare, respectively, with the author's values of 54.6 and 94.0.

In the section under the heading, "Replacing Rigid Bodies by Their Dynamic Reactions," attention is called to the fact that in vibration problems a rigid body must be replaced by its dynamic reactions rather than by its statical reactions at the points of support. A similar difference occurs when the energy method is used. If S_1 and S_2 denote the statical reactions, it is seen from Fig. 11 that the kinetic energy of the rigid machine during vibration is not correctly expressed by $\frac{S_1}{2\,g}\left(\frac{dy_1}{dt}\right)^2 + \frac{S_2}{2\,g}\left(\frac{dy_2}{dt}\right)^2$, but consists of the translatory energy of the mass concentrated in the gravity center, $\frac{S_1 + S_2}{2\,g}\left[\frac{d}{dt}\left(\frac{y_1\,l_b + y_2\,l_a}{l_a + l_b}\right)\right]^2$, and the rotatory energy, $\frac{J}{2\,g}\left[\frac{d}{dt}\left(\frac{y_2 - y_1}{l_a + l_b}\right)\right]^2$. Accordingly, in the denominator of the radicand of Eq. 86 the contribution of the machine loads $S_1\,\delta^2{}_1 + S_2\,\delta^2{}_2$ should be replaced by $(S_1 + S_2)\left(\frac{\delta_1\,l_b + \delta_2\,l_a}{l_a + l_b}\right)^2 + J\left(\frac{\delta_2 - \delta_1}{l_a + l_b}\right)^2$. However, in the example considered, the writer found the influence of this correction negligible. For the frequency of the fundamental mode no change was obtained in three significant figures, whereas the frequency of the second mode increased only from 54.3 to 54.6 (thus coinciding with Mr. Bleich's result).

HANS H. BLEICH,[9] M. ASCE.—Mr. Hauer's comment that the fundamental frequency of buildings can be found by simpler methods is correct for simple structures only, and the proposed method was developed just for the cases in which these simple methods fail. The fact that the frequencies in the example in the paper can be found by the Rayleigh method should not be misconstrued; in the "Synopsis," it is stated clearly that the example is a very simple system selected only to demonstrate all the steps of the method.

To indicate the type of structure for which the method was devised, Fig. 18 shows the framing of one half of the bay of a boilerhouse to which the method was applied. There were three bays, and the column spacing was close to 60 ft in both directions. The fans had two alternate speeds, 580 rpm and 725 rpm. Structures of this type have a number of frequencies close together in the vicinity of the fundamental frequency of the main girders, for this span between 200 rpm and 300 rpm. In addition, there are higher frequencies, which had to be kept high, out of resonance with the fans. The frequency analysis showed that, in order to achieve this, 57-in.-deep plate

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girders had to be used for the support of the fans (see Fig. 18). These girders are of short spans and, as a result, have very low stresses—less than 3,000 lb per sq in. for static loads. All other members of the floor have the sections determined from their static loads alone.

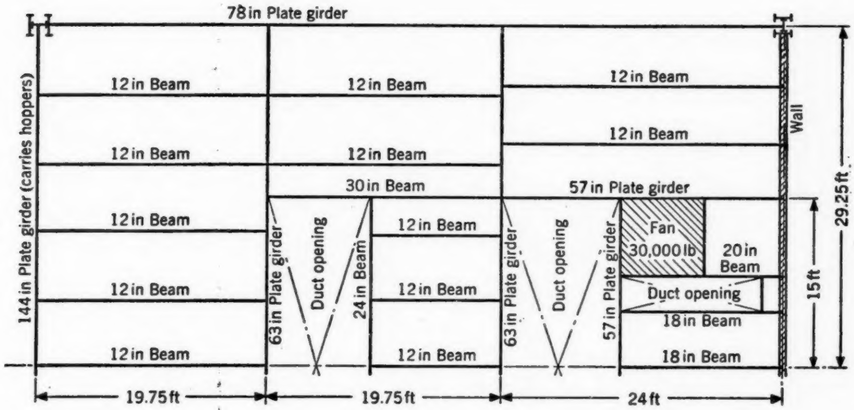

Fig. 18

In such cases, or in similar ones, the simple method suggested by Mr. Hauer would not be sufficient. In this connection it might be mentioned that recently developed methods of earthquake analysis require the knowledge not only of fundamental modes, but also of several higher modes, which can be determined by the method presented in the paper.